Tutorial 9

CSCI2110/MATH2080: Discrete Mathematics

6.1 - The Basics of Counting

6.2 - The Pigeonhole Principle

6.3 - Permutations and Combinations

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Question 1 - Bruteforcing Passwords

(Rosen 57/351) Suppose that a password for a computer system must have at least 8 but no more than 12 characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters: $*$, $>$, $<$, !, $+$, and $=$.

1. How many different passwords are available for the computer system?
2. How many of these passwords contain at least one occurrence of at least one of the six special characters?
3. Using your answer in part (a) and your crazy hacking skills, determine how long it takes a hacker to try every possible password, assuming it takes one nanosecond for a hacker to check each possible password.

Question 2 - Cool Applications of the Pigeonhole Principle

Apply the Pigeonhole Principle to the following facts. What conclusions can you draw?

1. Fact A: The human head has a maximum of 150,000 hairs.
   Fact B: The Durham Region has a population of 700,000.
2. Fact A: The total number of possible human fingerprints is estimated to be 64 billion.
   Fact B: There are approximately 8 billion humans on Earth.
3. Fact A: The oldest verified human lived 44,725 days.
   Fact B: On the average weekday, 208,700 people ride a GO train.

Question 3 - Proofs with Pigeonhole Principle

Prove one of the following statements (your choice):

1. From any 8 random numbers, you can select two whose difference is divisible by 7.
2. When you divide an integer by 19, the decimal expansion must eventually repeat.
3. There is a number in $\{8, 88, 888, 8888, 88888, ...\}$ that is divisible by 2021.