

Problem Set 3

Please complete and submit **any ONE** of the following problems. The deadline for submission is May 3, 2025 by 11:59 PM. Upload your submission here with the filename “ASP3-[Surname]”. Either typeset your solution using LaTeX or submit a clear PDF scan of your handwritten work. Clearly indicate which problem you are attempting to solve.

1. Let P be a convex polygon in \mathbb{R}^2 . Prove that the Voronoi diagram of the vertices of P has exactly one unbounded cell for each edge of P , and that each such cell is unbounded in a direction perpendicular to the corresponding edge.
2. Consider an affine transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ applied to a set of generator points S . Prove that the average number of edges per cell in a Voronoi diagram of n points is 6. Prove that the Voronoi diagram of $T(S)$ can be obtained by transforming the boundaries of the Voronoi diagram of S if and only if T is a composition of uniform scaling, rotation, and translation.
3. Prove that the average number of edges per cell in a Voronoi diagram of n points in general position in the plane is less than 6, using Euler’s formula for planar graphs.
4. Prove that weighted Voronoi cells are complex polytopes.
5. The City of Toronto is planning to place three new emergency response centers to minimize response time to five key neighborhoods. The locations of these neighborhoods (in kilometers, using a simplified coordinate system with the CN Tower as the origin) are:

Neighborhood	x	y
Downtown Core	0	0
North York	2	15
Scarborough	18	8
Etobicoke	-10	5
Liberty Village	-2	3

- (a) Draw the Voronoi diagram for these five neighborhood points on the plane.
- (b) If emergency response centers are to be placed at three of these neighborhood locations, which three locations would minimize the maximum distance any neighborhood is from the nearest center? Justify your answer using the Voronoi diagram.
- (c) If a new high-density housing development is established at High Park (coordinates (6,-2)), how would the Voronoi diagram from part (a) change? Which cells would be affected and how?
- (d) The Don Valley Parkway and Highway 401 represent major traffic corridors that affect emergency response times. How might you modify the standard Voronoi diagram to account for these transportation network? Describe conceptually

how the cells might change when time travel rather than Euclidean distance is considered.