**Problem Set 1**

*Please complete and submit* ***any ONE*** *of the following problems. The deadline for submission is May 3, 2025 by 11:59 PM. Upload your submission here with the filename "ASP1-[Surname]". Either typeset your solution using LaTeX or submit a clear PDF scan of your handwritten work. Clearly indicate which problem you are attempting to solve.*

1. Prove that if a transformation $A: \mathbb{R}^n \to \mathbb{R}^n$ preserves both the midpoints of line segments then $\mathrm{A}$ must be linear.
2. Prove that any affine transformation $A: X \to Y$ maps flats to flats. That is, if $F$ is a flat in $X$, then $A(F)$ is a flat $Y$.
3. Let $A$ be an affine space with direction space $V$. Prove that if $p_1, \dots, p_n$ are points in $A$, and if $c_1, \dots, c_n$ are scalars such that $\sum c_i = 1$, then $\sum c_i\, p_i$ is a vector in $V$.
4. Consider an affine transformation $A(x) = T(x) + b$ where $T$ is linear and $b$ is a fixed vector. Prove that if $A$ preserves parallelism of lines, then $T$ must be invertible.
5. Let $A$ and $B$ be two affine spaces with direction spaces $V$ and $W$, respectively. Prove that if $f: A \to B$ is a bijection that maps affine combinations to affine combinations (i.e., $f(\sum c_i\, p_i) = \sum c_i\, f(p_i)$) then $f$ must be an affine transformation.