Practice Problems

MATH2055: Advanced Linear Algebra Tutorial 10

Similarity and Diagonalization

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April 4, 2024

Question 1(a) - Generating Fibonacci Numbers

(Treil 4.2.10) Recall that the Fibonacci sequence,

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

is defined as follows. Put $\phi_0 = 0$ and $\phi_1 = 1$ and define $\phi_{n+2} = \phi_{n+1} + \phi_n$.
Find a 2×2 matrix A such that,

$$\begin{bmatrix} \phi_{n+2} \\ \phi_{n+1} \end{bmatrix} = A \begin{bmatrix} \phi_{n+1} \\ \phi_n \end{bmatrix}$$

Hint: Add the trivial equation $\phi_{n+1} = \phi_{n+1}$ to the Fibonacci relation $\phi_{n+2} = \phi_{n+1} + \phi_n$.

Question 1(b) - Generating Fibonacci Numbers

(Treil 4.2.10) Recall that the Fibonacci sequence,

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

is defined as follows. Put $\phi_0 = 0$ and $\phi_1 = 1$ and define $\phi_{n+2} = \phi_{n+1} + \phi_n$.
Diagonalize A and find a formula for A^n.

Question 1(c) - Generating Fibonacci Numbers

(Treil 4.2.10) Recall that the Fibonacci sequence,

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

is defined as follows. Put $\phi_0 = 0$ and $\phi_1 = 1$ and define $\phi_{n+2} = \phi_{n+1} + \phi_n$.
Noticing that,

$$\begin{bmatrix} \phi_{n+1} \\ \phi_n \end{bmatrix} = A^n \begin{bmatrix} \phi_1 \\ \phi_0 \end{bmatrix} = A^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

find a formula for ϕ_n. (You will need to compute an inverse and perform multiplication here.)

Question 1(d) - Generating Fibonacci Numbers

(Treil 4.2.10) Recall that the Fibonacci sequence,

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

is defined as follows. Put $\phi_0 = 0$ and $\phi_1 = 1$ and define $\phi_{n+2} = \phi_{n+1} + \phi_n$.

Show that the vector $\begin{bmatrix} \frac{\phi_{n+1}}{\phi_n} \\ 1 \end{bmatrix}$ converges to an eigenvector of A. What do you think, is it a coincidence?